Diffuseness of the Nuclear Surface from S-Wave Strength Functions*

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The S-wave strength function data is analyzed to find the diffuseness for the nuclei near $3S$ and $4S$ strength function giant resonances. The calculations are performed as a function of atomic weight, using a deformed optical-model potential for two values of the diffuseness parameter a in the Saxon-Woods potential, $a=0.52$ F and $a=0.30$ F. The comparison of the data with the calculations indicates the value of a to be (0.5 ± 0.1) F, for the nuclei of both the $3S$ and $4S$ giant resonances. Our result for the $4S$ giant resonance is in disagreement with that of Vogt. The reason for the disagreement is discussed.

IT was reported recently by Vogt[1,2] that the surface thickness a [cf. Eq. (4) below] of a target nucleus affects the surface penetrability of bombarding particles into the nucleus. This criterion was used to find the surface thickness for target nuclei in the region of $4S$ giant resonance of S-wave strength functions. In the method employed, the area under the experimentally measured $4S$ resonance was compared with that obtained by the spherical optical-model calculations for various values of a. However, the $4S$ giant resonance is known to split because of the deformations. The above area method, therefore, assumes an independence with respect to deformations. To test this assumption and to complete the work on the $3S$ giant resonance as well, calculations have been made at Brookhaven, using a deformed optical-model computer code written by Buck.[3]

To understand the role of a in the low-energy nuclear reactions (e.g., resonance cross sections or average cross sections described by the optical model) we begin by writing the compound nucleus cross sections for S-wave neutrons for a square-well complex potential:

$$V(r)=-(V_0+iW_0), \quad r\lesssim R \qquad (1)$$
$$=0, \quad r\gtrsim R$$

$$\sigma_c=4\pi\lambda\!\!\!^{-2}P_0\frac{-\mathrm{Im}f_0}{(P_0-\mathrm{Im}f_0)^2+(\mathrm{Re}f_0)^2}. \qquad (2)$$

f_0 is the logarithmic derivative of the wave function at the nuclear surface. P_0 is the penetrability of the S-wave neutron into the nuclear surface and is equal to kR where k is the wave number of the incident neutron and R the nuclear radius.

From reaction theory one can write the same cross section in terms of the above-mentioned quantity strength function S_0 as

$$\sigma_c=4\pi^2\lambda\!\!\!^{-2}P_0S_0. \qquad (3)$$

Note that the above is true only in the limit of low energies with which we are concerned for the time being.

A comparison of Eqs. (2) and (3) will indicate a resonance behavior of S_0 when plotted as a function of atomic weight for a fixed energy. For the case of a diffuse Saxon-Woods potential,

$$V(r)=-(V_0+iW_0)/(1+e^{(r-R)/a}), \qquad (4)$$

Vogt points out that Eq. (3) still remains valid with corresponding P_0 changed to $P_a=P_0c(a)$, where c is independent of energy and depends only on the surface thickness a.

If we define a new quantity $S_a=S_0c(a)$, then $\sigma_c=4\pi^2\lambda\!\!\!^{-2}P_0S_a$ is identical in form with Eq. (3). An integration of S_a over the giant resonance differs from that of S_0 also by $c(a)$, except for a small radius dependence. A comparison of the area under measured strength functions S_a with that calculated for various values of a then allows one to find the surface thickness, a, and the quantity, $c(a)$, the additional surface penetration due to diffuseness of the nuclear potential.

Assuming that $c(a)$ is independent of the particular model (deformed or undeformed, volume absorption or surface absorption) Vogt found that for the $4S$ giant resonance nuclei $a=(0.31\pm0.06)$F. Extending his calculations for the $3S$ giant resonance, where nuclei are spherical and above assumption is not needed, we found the thickness $a=(0.5\pm0.1)$F. This would have meant a change in diffuseness a with atomic weight. Consequently, calculations were made for the $4S$ giant resonance using a deformed optical-model code,[3] for various values of the surface thickness a. The real part of potential was changed so that the calculated peaks of S-wave strength functions fall at the positions observed experimentally. The deformation parameter β for each nucleus was obtained from the Coulomb excitation studies.[4] Experimentalists in the past have defined a quantity, $\langle\Gamma_n^0\rangle/D=RS_a\times10^{-4}/0.2276$, where $\langle\Gamma_n^0\rangle$ is the average neutron width at 1 eV and D is the average level spacing. It is this quantity which is calculated below for comparison with experiments.

Figure 1 shows the experimental data[5] and calculations for a Saxon-Woods potential with $V_0=49$ MeV,

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[1] E. Vogt, Phys. Letters **1**, 84 (1962).
[2] E. Vogt, Rev. Mod. Phys. **34**, 723 (1962).
[3] B. Buck, Phys. Rev. Letters **8**, 444 (1962).
[4] B. Buck (private communication), compilation of deformation parameters.
[5] J. Gibbons (private communication), recent compalition of S-wave strength functions.

$W=3.12$ MeV, $R=1.25\ A^{1/3}$F, and $a=0.52$ F. Figure 2 shows the same calculations for a diffuseness $a=0.3$ F as obtained by Vogt. In both of these cases a vibrational model calculation is performed (using the same code) for the nuclei of the $3S$ giant resonance which are not deformed. From Figs. 1 and 2 one finds that a diffuseness of 0.5 F is a good fit, while 0.3 F is very unreasonable.

From subsequent calculations, a diffuseness of (0.5 ± 0.1)F was deduced. This implies that Vogt's assumption that the additional surface penetration factor $c(a)$ is independent of the model is not correct. Alternatively, the sum rule that the area under a giant resonance is independent of the deformation is incorrect by the same token. To test this, a calculation was made for a spherical potential for the $4S$ giant resonance, using the same diffuseness as for the deformed model. The ratio of the areas under two curves differs by about 40%.

It is therefore concluded that the diffuseness of the nuclear surface is (0.5 ± 0.1) F, in agreement with that obtained by high-energy electron-scattering measurements.[6] Therefore, there is no need for introducing any new polarization phenomenon[1] as suggested by Vogt to explain the then existing difference between the diffuseness obtained by electron scattering and that obtained by neutron scattering. The area sum rule is also found to depend on the deformations. The effect of the surface absorption on the sum rule and the detailed shape of S-wave strength functions will be described in another paper. It may now be pointed out that the area sum rule is found to depend on the shape of the imaginary part of the potential as well.

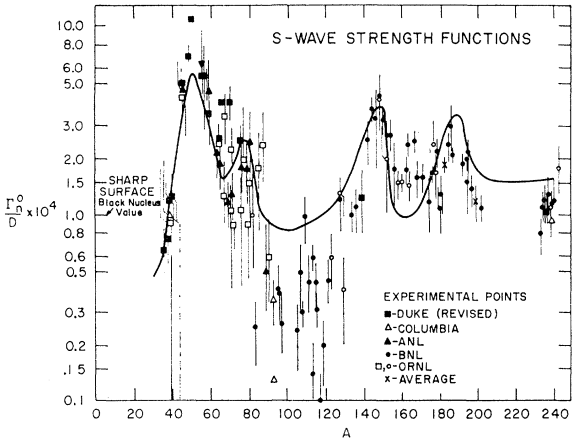

FIG. 1. The neutron strength function data of Ref. 5 as a function of atomic weight and optical-model computations of strength functions with a deformed code for the parameters: $V_0=49$ MeV, $W_0=3.12$ MeV, $R=1.25\ A^{1/3}$ F, and $a=0.52$ F.

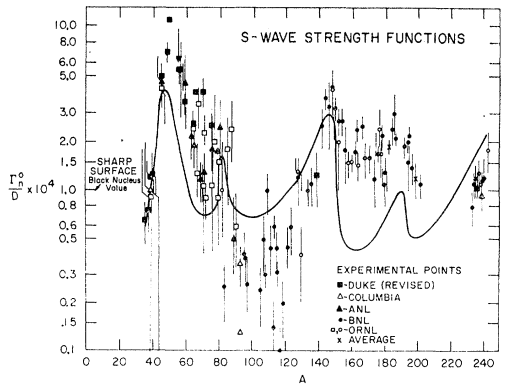

FIG. 2. The neutron strength function data of Ref. 5 as a function of atomic weight and optical-model computations of strength functions with a deformed code for the parameters: $V_0=49$ MeV, $W_0=3.12$ MeV, $R=1.25\ A^{1/3}$ F, and $a=0.30$ F.

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[6] D. G. Ravenhall, Rev. Mod. Phys. **30**, 430 (1958).